Homework #9

(Physics 230A, Spring 2006)

Due 10:10 AM, June 7, 2006 (before the Wed. class)

1. (10 pts) Let $|\Psi_T\rangle$ be a state which contains transverse photons only. Let

$$|\Psi'_T\rangle = \left\{1 + c\left[a_3^\dagger(\vec{k}) - a_0^\dagger(\vec{k})\right]\right\}|\Psi_T\rangle\,, \tag{1}$$

where c is some constant. Show that replacing $|\Psi_T\rangle$ by $|\Psi'_T\rangle$ corresponds to a gauge transformation, *i.e.*,

$$\langle\Psi'_T|\,A^\mu(x)\,|\Psi'_T\rangle = \langle\Psi_T|\,[A^\mu(x) + \partial^\mu\Lambda(x)]\,|\Psi_T\rangle\,, \tag{2}$$

where

$$\Lambda(x) = \left(\frac{2}{V\omega_{\vec{k}}^3}\right)^{1/2}\mathrm{Re}(ice^{-ik\cdot x}). \tag{3}$$

2. (10 pts) As an explicit check of Wicks theorem for four scalar fields, show explicitly that

$$T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\} =: \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) + \sum \text{all contractions} :, \tag{4}$$

assuming $x_2^0 > x_1^0 > x_3^0 > x_4^0$.