Homework #7

(Physics 230A, Spring 2006)

Due 5 PM, Thu., May 25, 2006

1. (10 pts) Show that the $S^{\mu\nu} \equiv \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ matrices obey the commutation relationship required for them to be a representation of the Lorentz group generator algebra:

$$[S^{\mu\nu}, S^{\rho\sigma}] = i[g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}]. \tag{1}$$

2. (10 pts) Derive the expression for the charge operator $Q = q\sum_{r,\vec{p}}[N_r(\vec{p}) - \overline{N}_r(\vec{p})]$ for the Dirac field as given in the lecture notes. Remember to use the normal ordering in the last step.

3. (10 pts) Show that

$$\{\psi^\pm_\alpha(x), \overline{\psi}^\mp_\beta(y)\} = i(i\not\partial + m)_{\alpha\beta}\Delta^\pm(x-y). \tag{2}$$

You need the identities derived in the 5/15 class,

$$\sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) = \not p + m, \quad \sum_r v_r(\vec{p})\bar{v}_r(\vec{p}) = \not p - m. \tag{3}$$

These identities are important for calculating scattering matrix elements in the future and they have not been derived in the lecture notes yet, but you can find them in Peskin & Schroeder, p.49, eqs.(3.66), (3.67), or Mandl & Shaw, Appendices A.4, A.5. (Mandl & Shaw use a different normalization, so they get $(\not p \pm m)/(2m)$.)