Homework #3

(Physics 230A, Spring 2006)

Due 10:10 AM, April 26, 2006 (before the Wed. class)

Use $\hbar = c = 1$ in both problems for convenience.

1. (10 pts) The Dirac equation for an electron in the presence of external electromagnetic fields is

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi = 0. \tag{1}$$

(a) Multiplying $[i\gamma^\nu(\partial_\nu + ieA_\nu) + m]$ on the left, derive the second order equation,

$$\left[(\partial_\mu + ieA_\mu)(\partial^\mu + ieA^\mu) + \frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} + m^2\right]\psi = 0. \tag{2}$$

(b) Consider an electron in a uniform and constant magnetic field $\vec{B} = B\hat{z}$ along the z-axis ($\vec{E} = 0$). Assuming that the gyromagnetic ratio $g_s = 2$, show that the energy eigenstates are given by $E^2 = m^2 + p_z^2 + 2n|e|B$ where n is an integer.
(Hint: You can pick a convenient gauge, $\vec{A} = B\frac{x}{2}\hat{y} - B\frac{y}{2}\hat{x}$. Check that it gives the constant magnetic field described above. Then you can use it in the second order equation in (a). After separating the x, y directions from the z direction, you find the equation in the x, y directions looks like the 2-dimensional harmonic oscillator plus the angular momentum operator. Try to define appropriate creation and annihilation operators which satisfy the canonical commutation relation, and write the equation in terms of the creation and annihilation operators. At that point you should be able to read off the eigenvalues.)

2. (10 pts) In class we showed that a charged Dirac particle with an anomalous gyromagnetic ratio $g_s = 2(1 + a)$ can be described by a modified Dirac equation,

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m + k\sigma_{\mu\nu}F^{\mu\nu}]\psi = 0, \tag{3}$$

where $k = -\frac{ae}{4m}$. Consider such a particle moving in a uniform and constant magnetic field $\vec{B}$ ($\vec{E} = 0$). Show that the difference in the spin precession and orbital frequencies is proportional to $g_s - 2$.
(Hint: First find the hamiltonian and calculate $\frac{d\vec{\Sigma}}{dt}$ and $\frac{d\vec{\pi}}{dt}$, where $\vec{\pi} = \vec{p} - e\vec{A}$ is the gauge invariant combination of the momentum. (In a gauge where $\vec{A}$ is time-independent, $\frac{d\vec{\pi}}{dt} = \frac{d\vec{p}}{dt}$.). They correspond to the rates of the changing in the directions of the spin and the momentum respectively. Since they are periodic in a constant magnetic field, the frequency of $\frac{d(\vec{\Sigma}\cdot\vec{\pi})}{dt}$ is the difference between the spin precession and the orbital motion.)