As we discussed in class, a positive energy Dirac spinor of momentum $\vec{p}$ in the Pauli-Dirac representation is

$$u = \sqrt{E + m} \left( \frac{\chi_s}{\sigma \cdot \vec{p}} \chi_s \right). \quad (1)$$

1. (10 pts) We would like to find the rotation transformation (in the passive sense),

$$S = \mathbf{1} \cos \frac{\theta}{2} + i \hat{n} \cdot \vec{\Sigma} \sin \frac{\theta}{2}$$

such that in the new frame the spinor is moving in the $+\hat{z}'$ direction.

(a) First by geometric reasoning find what $\theta$ and $\hat{n}$ should be and express them in terms of $\vec{p}$ and $\hat{z}$ using what you learned from the vector space.

(b) Then check that $S u$ is indeed a spinor moving in the $+\hat{z}'$ direction. (Of course, the direction of the spin is also transformed.)

2. (10 pts)

(a) Find out what the spinor $u$ look like in the Weyl representation by performing a similarity transformation.

(b) In Peskin and Schroeder (page 46), a spinor in the Weyl representation is written as

$$\left( \begin{array}{c} \sqrt{p \cdot \sigma} \chi_s \\ \sqrt{p \cdot \bar{\sigma}} \chi_s \end{array} \right) \quad \text{where} \quad p \cdot \sigma \equiv EI - \vec{p} \cdot \vec{\sigma}$$

$$p \cdot \bar{\sigma} \equiv EI + \vec{p} \cdot \vec{\sigma}. \quad (2)$$

Show that the result in (a) is equivalent to this expression.

(Hint: The square root of a semi-positive-definite hermitian matrix is defined by the square roots of the eigenvalues. If $H = U^{-1}H_D U$ where $H_D$ is a diagonal matrix, then

$$\sqrt{H} \equiv U^{-1}\sqrt{H_D} U$$

where $\sqrt{H_D}$ is the diagonal matrix with the diagonal elements given by the square roots of the diagonal elements of $H_D$.)