Homework #2

(Physics 230A, Spring 2006)

Due 10:10 AM, April 19, 2006 (before the Wed. class)

As we discussed in class, a positive energy Dirac spinor of momentum $\vec{p}$ in the Pauli-Dirac representation is

$$u = \sqrt{E+m} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \end{pmatrix}. \tag{1}$$

1. (10 pts) We would like to find the rotation transformation (in the passive sense), $S = \mathbf{1}\cos\frac{\theta}{2} + i\hat{n}\cdot\vec{\Sigma}\sin\frac{\theta}{2}$ such that in the new frame the spinor is moving in the $+\hat{z}'$ direction.

(a) First by geometric reasoning find what θ and $\hat{n}$ should be and express them in terms of $\vec{p}$ and $\hat{z}$ using what you leared from the vector space.

(b) Then check that $S\,u$ is indeed a spinor moving in the $+\hat{z}'$ direction. (Of course, the direction of the spin is also transformed.)

2. (10 pts)

(a) Find out what the spinor u look like in the Weyl representation by performing a similarity transformation.

(b) In Peskin and Schroeder (page 46), a spinor in the Weyl representation is written as

$$\begin{pmatrix} \sqrt{p\cdot\sigma}\chi_s \\ \sqrt{p\cdot\bar{\sigma}}\chi_s \end{pmatrix} \quad \text{where} \quad \begin{array}{l} p\cdot\sigma \equiv E\boldsymbol{I} - \vec{p}\cdot\vec{\sigma} \\ p\cdot\bar{\sigma} \equiv E\boldsymbol{I} + \vec{p}\cdot\vec{\sigma} \end{array}. \tag{2}$$

Show that the result in (a) is equivalent to this expression.

(Hint: The square root of a semi-positive-definite hermitian matrix is defined by the square roots of the eigenvalues. If $H = U^{-1}H_D U$ where H_D is a diagonal matrix, then $\sqrt{H} \equiv U^{-1}\sqrt{H_D}U$ where $\sqrt{H_D}$ is the diagonal matrix with the diagonal elements given by the square roots of the diagonal elements of H_D.)