Homework #1

(Physics 230A, Spring 2006)

Due 10:10 AM, April 12, 2006 (before the Wed. class)

1. (10 pts) The Klein-Gordon equation in the presence of the electromagnetic field is

$$(i\hbar\partial_t - e\phi)^2\psi = c^2(-i\hbar\vec{\nabla} - \frac{e}{c}\vec{A})^2\psi + m^2c^4\psi = 0.$$

Find out how the wavefunction ψ should transform under the gauge transformation $\vec{A} \to \vec{A} - \vec{\nabla}\chi$, $\phi \to \phi + \frac{1}{c}\partial_t\chi$ to keep the Klein-Gordon equation invariant.

2. (10 pts) The Pauli-Dirac representation of the gamma matrices are

$$\gamma^0_{\rm PD} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad \gamma^i_{\rm PD} = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \; i = 1, 2, 3, \quad \gamma^5_{\rm PD} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}.$$

The Majorana representation of the gamma matrices are obtained by the transformation matrix S, $S\,\gamma^\mu_{\rm PD}\,S^{-1} = \gamma^\mu_{\rm M}$, with $S = \frac{1}{\sqrt{2}}\gamma^0_{\rm PD}(1 + \gamma^2_{\rm PD})$.

(a). Compute $\gamma^\mu_{\rm M}$ and $\gamma^5_{\rm M}$ and verify that all gamma matrices are imaginary in the Majorana representation.

(b). Using the Majorana representation, show that if a particle ψ satisfying the Dirac equation has charge q, that is $i\gamma^\mu(\partial_\mu + i\frac{q}{\hbar}A^\mu)\psi - \frac{mc}{\hbar}\psi = 0$, then ψ^* represents a particle with charge $-q$.